

b) Encuentra la derivada para $x \in [0, \infty)$ de la función $f(x) = \sqrt{x}$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot 1 = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right)$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{x}}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{1}{(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$