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f) Si $g(a) \neq 0$ entonces $\left(\frac{f(a)}{g(a)}\right)' = \frac{f'(a)g(a) - f(a)g'(a)}{(g(a))^2}$

$$\lim_{h\to 0} \frac{1}{h}\left(\frac{f(a+h)}{g(a+h)} - \frac{f(a)}{g(a)}\right) = \lim_{h\to 0} \frac{1}{h}\left(\frac{g(a)f(a+h) - f(a)g(a+h)}{g(a+h)g(a)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{g(a)f(a+h) + 0 - f(a)g(a+h)}{g(a+h)g(a)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{g(a)f(a+h) - g(a)\cdot f(a) + g(a)f(a) - f(a)g(a+h)}{g(a+h)g(a)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{g(a)(f(a+h) - f(a)) + f(a)(g(a) - g(a+h))}{g(a+h)g(a)}\right)$$

$$= \lim_{h\to 0} \frac{g(a)(f(a+h) - f(a))}{h(g(a+h)\cdot g(a))} + \lim_{h\to 0}\left(\frac{f(a)(g(a) - g(a+h))}{h(g(a+h)\cdot g(a))}\right)$$

$$= g(a)\lim \frac{f(a+h) - f(a)}{h}\cdot\frac{1}{g(a+h)\cdot g(a)} - f(a)\lim_{h\to 0}\frac{g(a) - g(a+h)}{h}\cdot\frac{1}{g(a+h)\cdot g(a)}$$

$$= g(a)\cdot f'(a)\cdot\frac{1}{(g(a))^2} - f(a)\cdot g'(a)\cdot\frac{1}{(g(a))^2}$$

$$= \frac{g(a)f'(a) - f(a)g'(a)}{(g(a))^2} = \frac{f'(a)g(a) - f(a)g'(a)}{(g(a))^2}$$