– $f(x) = \operatorname{sen} x \cdot \cos x$ ¿Cuánto vale la derivada.

$$f'(x) = (\operatorname{sen} x \cdot \cos x)' = (\operatorname{sen} x)' \cdot \cos x + \operatorname{sen} x \cdot (\cos x)'$$

Lo que nos falta ver o saber ¿cuánto vale $(\operatorname{sen} x)'$ y $(\cos(x))'$?

Por definición

$$(\operatorname{sen} x)' = \lim_{h\to 0} \frac{\operatorname{sen}(x+h) - \operatorname{sen}(x)}{h} = \lim_{h\to 0} \frac{\operatorname{sen} x \cos h + \cos x \operatorname{sen} h - \operatorname{sen} x}{h}$$

$$= -\lim_{h\to 0} \frac{(\operatorname{sen} x)(1-\cos h)}{h} + \lim_{h\to 0} \frac{\cos x \operatorname{sen} h}{h} = \underline{\cos x} \,//$$

$(\operatorname{sen} x)' = \cos x$

$\operatorname{sen}(x+h) = \operatorname{sen} x \cdot \cos h + \cos x \operatorname{sen} h$ ① ver apendice de Trigonometria.

$\lim_{h\to 0} \frac{\operatorname{sen} h}{h} = 1$ y $\lim_{h\to 0} \frac{1-\cos h}{h} = 0$ ① ver apendice de Trigonometria y limites

$$(\cos x)' = \lim_{h\to 0} \frac{\cos(x+h) - \cos(x)}{h} = \lim_{h\to 0} \frac{\cos x \cos h - \operatorname{sen} x \operatorname{sen} h - \cos x}{h}$$

$\cos(x+h) = \cos x \cos h - \operatorname{sen} x \operatorname{sen} h$ ①

$$= \lim_{h\to 0} \frac{\cos x(\cos h - 1) - \operatorname{sen} x \operatorname{sen} h}{h}$$

$$= \cos x \lim_{h \to 0} \frac{(\cos h - 1)}{h} - \operatorname{sen} x \lim_{h \to 0} \frac{\operatorname{sen} h}{h}$$

$$= \cos x (0) - \operatorname{sen} x (1) = -\operatorname{sen} x$$

Por lo cuál

$$(f(x))' = (\operatorname{sen} x \cos x)' = (\operatorname{sen} x)' \cos x + \operatorname{sen} x (\cos x')$$

$$= \cos x \cos x + \operatorname{sen} x (-\operatorname{sen} x)$$

$$= \cos^2 x - \operatorname{sen}^2 x$$

$$= \cos^2 x - (1 - \cos^2 x)$$

$$= \cos^2 x - 1 + \cos^2 x$$

$$= 2 \cos^2 x - 1$$