Demostración

Usando el teorema de la derivada de la función inversa

Tenemos que

$$f \circ f^{-1}(x) = x$$

$$(f \circ f^{-1}(x))' = (x)'$$

$$f'(f^{-1}(x)) \cdot (f^{-1})'(x) = 1$$

$$(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}$$

a) $$(\arcsin x)' = \frac{1}{(\operatorname{sen}'(\operatorname{arc}(\operatorname{sen}(x)))}$$

$$= \frac{1}{\cos(\operatorname{arc}\operatorname{sen}(x))}$$

tenemos que usando el dibujo

$$\operatorname{sen}\theta = \frac{x}{1} = x$$

$$\cos\theta = \sqrt{1-x^2}$$

$$(\operatorname{sen}(\operatorname{arc\,sen}(x)))^2 + ((\cos(\operatorname{arc\,sen} x))^2 = 1$$

$$x^2 + \cos(\operatorname{arc\,sen} x)^2 = 1$$

$$\cos(\operatorname{arc\,sen} x)^2 = 1 - x^2$$

$$\cos(\operatorname{arcsen} x) = \sqrt{1-x^2}$$

$$\therefore \quad (\operatorname{arc\,sen} x)' = \frac{1}{\sqrt{1-x^2}}$$

b) $$(\operatorname{arc}\cos(x))' = \frac{1}{-\operatorname{sen}(\operatorname{arc}(\cos x))}$$

$$\operatorname{sen}(\operatorname{arc}(\cos x))^2 + \cos(\operatorname{arc}(x))^2 = 1$$

$$\operatorname{sen}(\operatorname{arc}(\cos x))^2 + x^2 = 1$$

$$\operatorname{sen}(\operatorname{arc}(\cos(x)))^2 = 1 - x^2$$

$$\operatorname{sen}(\operatorname{arc}(\cos x))^2 = 1 - x^2$$

$$\operatorname{sen}(\operatorname{arc}(\cos x))^2 = \sqrt{1-x^2}$$

$$\therefore (\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

c) $$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\arctan x)' = \frac{1}{(\tan(\arctan x))'}$$

$$= \frac{1}{\sec^2(\arctan x)} = \frac{1}{\tan(\arctan x)^2+1}$$

$$= \frac{1}{x^2+1}$$

$$= \frac{1}{1+x^2}$$

Tenemos

$$\operatorname{sen}^2 u + \cos^2 u = 1$$

$$\frac{\operatorname{sen}^2 u}{\cos^2 u} + \frac{\cos^2 u}{\cos^2 u} = \frac{1}{\cos^2 u}$$

$$\tan^2 u + 1 = \frac{1}{\cos^2 u} = \sec^2 u$$